in Sect 1 and (-1) for Sect. 2. Overall (-2) whole paper



Rossmoyne SHS Mathematics Department

MATHEMATICS 3A MAT

Semester 1 2011 EXAMINATION

NAME:

TEACHER: Belonogoff Benko Fletcher Goh Longley Pui

(Circle one name) Rigelsford Robinson Tanday Tay White Whyte

Section One: Calculator-free

Time allowed for this section

Reading time before commencing work: 5 minutes Working time for this section: 50 minutes

Material required/recommended for this section

To be provided by the supervisor This Question/Answer Booklet Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	
Section One: Calculator-free			50	40	
Section Two: Calculator-assumed			100	80	
				120	

Instructions to candidates

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- 2. Answer the questions in the spaces provided.

Section One: Write answers in this Question/Answer Booklet. Answer all questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you do not use pencil, except in diagrams.

- 3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
- 4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

QUESTION	MARKS AVAILABLE	STUDENT MARK
1	5	
2	3	
3	2	
4	3	
5	7	
6	5	
7	5	
8	3	
9	2	
10	5	
TOTAL	40	

SECTION A

NO CALCULATORS PERMITTED FOR THIS SECTION

TIME

50 minutes

AVAILABLE MARKS 40 marks

Question 1 (1,2,1,1 marks)

- (a) Determine the possible value/s of x in each of the following:
- (i) |x| = -10

No solution V

RW

(ii) |x-3| = 9

n =12

 $\kappa = -6$

- (b) Solve the following equations:
- (i) $\left(3^7 \times 3^{-4}\right)^3 = 3^{2n+1}$

n = 4

RW

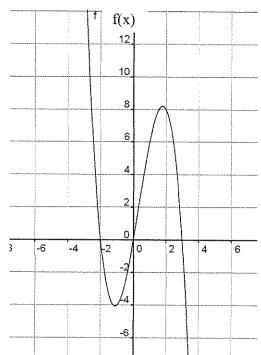
(ii) $\frac{2^5 \times 2^3}{2^{11}} = 2^{-n}$

n=3/2 RM

4

Question 2 (2,1 marks)

(a) Determine the equation of the function, f(x), below:



 $f(\alpha) = - \left(\alpha + 2 \right) (\alpha - 3) \sqrt{(\alpha - 3)} \sqrt{$

-1 for each error/omission

(b) Hence, solve f(x) = 0.

 $\chi = 0$ or $\chi = -2$ or $\chi = -3$

Question 3 (2 marks)

(a) A sequence is defined recursively as:

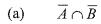
$$T_{n+3} = T_{n+2} + 2T_{n+1} - T_n$$
 where $T_1 = 7$, $T_2 = 10$ and $T_3 = 6$

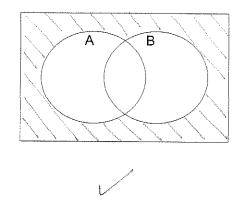
Determine the fourth and fifth terms of the sequence.

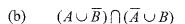
Let
$$n=1$$
 $T_4 = T_3 + 2T_2 - T_1$ () = $6 + 2(10) - 7$ = 19

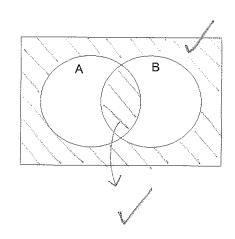
Let
$$n=2$$
 $T_5=T_4+2T_3-T_2$ RW
= $19+2(6)-10$

Question 4 (1,2 marks) Shade the following descriptions on the Venn Diagrams provided.





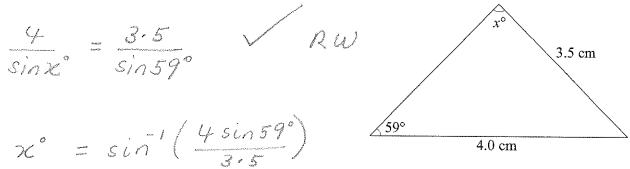




Question 5 (1,1,1,1,1,1,1 marks)

For the diagrams on the right (below),

- (a)
- Write an equation that will allow you to solve for the value of x. (Do NOT solve it.) (i)



(Diagram not drawn to scale)

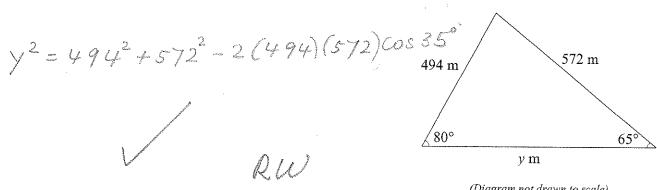
Given that $x^{\circ} \approx 78^{\circ}$ is a solution to the equation in (i) above. State another value of x° , in (ii) degrees, (if any) that will satisfy the same given measurements? Explain why you were able or not able to find another value of x° in this situation.

x=101.6°, since 101.6°+59°<180° :. possible to construct another \(\Delta \)

(for satisfying these given measurements.

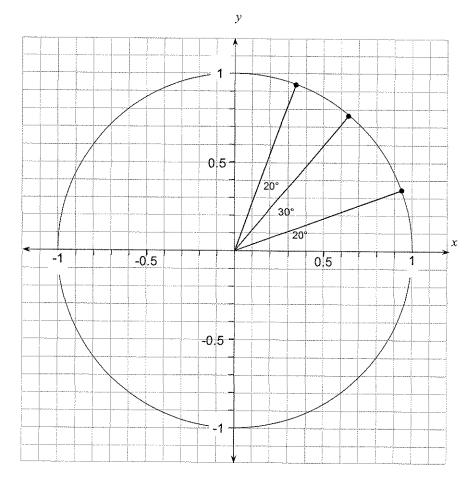
number only)

(b) Write an equation that will allow you to solve for y. (Do NOT solve it).



(Diagram not drawn to scale)

(c) Use the unit circle diagram below to determine the approximate value of each of the following trigonometric ratios:



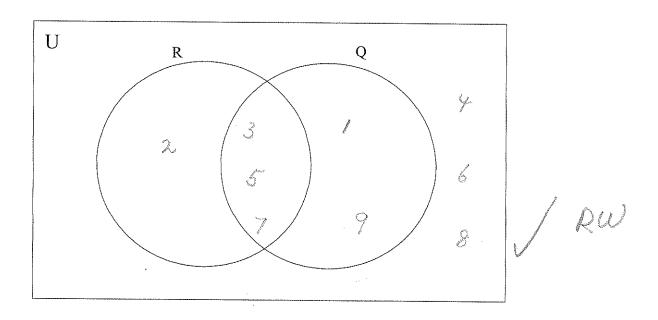
Accept to.01° difference

(i)
$$\sin 20^{\circ} \approx 0.34$$

Question 6 (5 marks)

If $Q = \{odd \ numbers\}$, $R = \{prime \ numbers\}$ and U (the universal set) is the set of digits, :

(a) Complete the Venn diagram below to show this information.



(b) Determine:

(i)
$$\overline{R} = \{ 1, 4, 6, 8, 9 \}$$

(ii)
$$Q \cap R = \{3, 5, 7\}$$

(iii)
$$\overline{Q \cup R} = \{4, 6, 8\}$$

(iv)
$$\overline{Q} \cap \overline{R} = \{4,6,8\}$$

Question 7 (1,1,1,1,1 marks)

In order to finish her driver's licence test, Dolly has to answer three 'true' or 'false' questions.

Use a tree diagram to show all the possible ways Dolly could answer the questions. (a) (Use T = true and F = false)

FTT (accept tree FTT (accept tree FTT (accept tree FTT (accept tree CETT (accept tree

If she gets more than one wrong, she fails the test. If the correct answer for each question was 'True' (T), determine the probability of:

(b) getting all three correct

Follow through

passing the test (c)

(d) passing the test if she knew the answer to the first question

(e) getting one question wrong given that Dolly passed the test

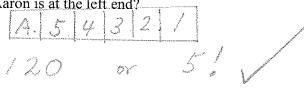
Question 8 (1,1,1 marks)

Six members of the Math Quest Prize Winners have to arrange themselves in line for a photograph. The members are: Aaron, Bree, Charlie, Dylan, Evelyn and Freddy.

RW

How many arrangements are there in which: (You may leave your answer in factorial form.)

(a) Aaron is at the left end?



(b) Aaron is at the left end or Freddy is at the right end?



(c) Aaron and Freddy are not next to each other?



Question 9 (2 marks)

Holly scores 26, 35 and 19 marks in three assessments in mid term, respectively. If the mean and standard deviation in each of these assessments are as given below, determine if she has improved in her performance? Justify your answer.

Assessment 1:	mean 34	standard deviation 8
Assessment 2:	mean 32	standard deviation 6
Assessment 3:	mean 17	standard deviation 2.5

Z score (assessment 1) =
$$\frac{26-34}{8}$$
 = -1

(assessment 2) = $\frac{35-32}{6}$ = 0.5

(assessment 3) = $\frac{19-17}{2.5}$ = 0.8

From Z scores above, Holly has improved

Question 10 (3,2 marks)

(a) Determine the median, lower quartile and upper quartile for the following set of scores:

20, 29, 17, 28, 27, 17, 30, 23, 30, 28, 21, 17

Re-arranging 17, 17, 17, 20, 21, 23, 27, 28, 28, 29, 30, 30

Median = 25 V

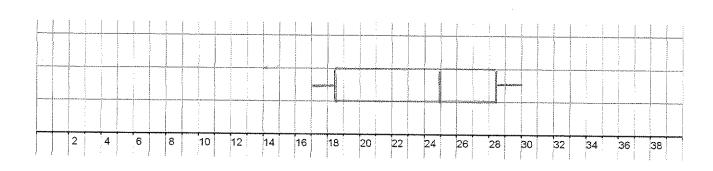
L.Q. = 18.5 V U.Q. = 28.5

RW

(b) Draw the box plot for the set of scores in (a) above on the grid below.

*Lowest values L.Q. }

correctly plotted & & u.Q. }



NB: Max deduction (-1 for units)
in Sect. 2



Rossmoyne SHS Mathematics Department

MATHEMATICS 3AB MAT

Semester 1 2011 EXAMINATION

NAME:

TEACHER:

Belonogoff

Benko

Fletcher

Goh

Longley

Pui

(Circle one name)

Rigelsford

Robinson Tanday

Tay

White

Whyte

Section Two: Calculator-assumed

Time allowed for this section

Reading time before commencing work:

10 minutes

Working time for this section:

100 minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to

three calculators satisfying the conditions set by the Curriculum Council for this

examination

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QUESTION	MARKS AVAILABLE	STUDENT MARK
1	6	
2	3	
3	4	
4	8	
5	8	
6	6	
7	12	
8	8	
9	5	
10	6	
11	3	
12	4	
13	7	
TOTAL	80	

SECTION B

APPROVED CALCULATORS PERMITTED FOR THIS SECTION

TIME:

100 minutes

MARKS ALLOCATED 80 marks

Question 1. (1,1,2,1,1 marks)

Mr and Mrs Smith have borrowed \$250 000 to purchase a unit for their son. They would like to repay their loan at \$2 500 per month. Below is a monthly summary for the first 18 months of the repayment of the loan.

Month	Balance at start of month	Interest	Repayment	Balance at end of month
1	250 000.00	1 250.00	2 500.00	248 750.00
22	248 750.00	1 243.75	2 500.00	247 493.75
3	247 493.75	1 237.47	2 500.00	246 231.22
4	246 231.22	1 231.16	2 500.00	244 962.37
5	244 962.37	1 224.81	2 500,00	243 687.19
6	243 687.19	1 218.44	2 500.00	242 405.62
7	242 405.62	1 212.03	2 500.00	241 117.65
8	241 117.65	1 205.59	2 500.00	239 823.24
9	239 823.24	1 199.12	2 500.00	238 522.36
10	238 522.36	1 192.61	2 500.00	237 214.97
11	237 214.97	1 186.07	2 500.00	235 901.04
12	235 901.04	1 179.51	2 500.00	234 580.55
13	234 580.55	1 172.90	2 500.00	233 253.45
14	233 253.45	1 166.27	2 500.00	231 919.72
15	231 919.72	1 159.60	2 500.00	230 579.32
16	230 579.32	1 152.90	2 500.00	229 232.21
17	229232-21	1146.16	2 500.00	227878.37
18	227878.37	1139.39	2 500.00	226517.77

(i) What is the monthly interest rate?

(ii) What is the yearly interest rate?

- (iii) Complete the last 2 rows of the table (round your answers to the nearest cent).
- (iv) Write a recursive formula to determine the amount owing at the start of each month.

(v) How long (in months) will it take them to completely pay off the loan?

· Commence of the control of the con

Question 2. (1,1,1 marks)

Each year the value of a car depreciates by 15% of its value at the beginning of that year. If the car is initially worth \$20 000, determine:

(a) an expression for the value, V, of the car after n years.

V = 20000 (0.85)" /

Follow through

(b) the value of the car after 10 years.

\$3937.49

(c) in how many years' time will the value of the car fall below \$1000? (to the nearest year.)

19 years

Question 3 (2,1,1 marks)

The resistance, R ohms, to the flow of electricity in a wire varies inversely as the area of cross-section of the wire, A.

(a) Given that when $A = 0.15 \text{ cm}^2$, R = 0.24 ohms. Write an equation connecting the two variables R and A.

- (b) Determine
- (i) R when $A = 0.07 \text{ cm}^2$

Follow through
(providing R&A in inverse relationship)

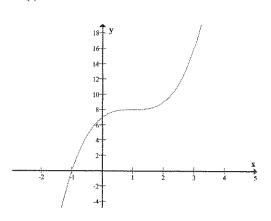
R = 0.5/43 ohms

(ii) A when R = 0.45 ohms.

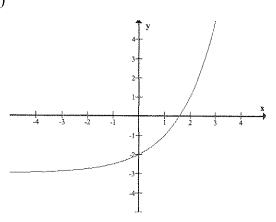
A = 0.08 cm2

Question 4 (4,4 marks)

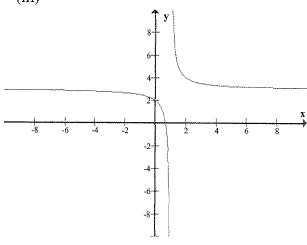
(i)



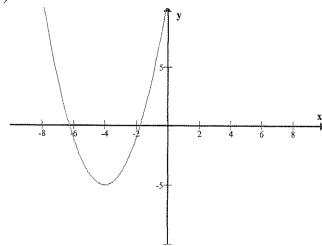
(ii)



(iii)



(iv)



Match the graphs of the above curves with the following equations: (a)

$$y = \sqrt{x+a}$$

$$y = \frac{1}{x+b} + c$$
 $y = (x+d)^3 + e$ $y = f^x + g$ $y = (x+h)^2 + j$

$$y = (x + d)^3 + e$$

$$y = f^x + g$$

$$y = (x + h)^2 + j$$

Write the equation in the table provided below.

Graph (i)	Graph (ii)	Graph (iii)	Graph (iv) y=(x+h)+j	
y-(x+d)+e	Yafxtq	Y= 56+6+6		
\ marketing	and the second second	* And the state of	1 amount	

(b) Hence, find the unknowns (*i.e.* a, b, c, d, e, f, g, h, j) related to each equation respectively and write your answer in the table provided below. (Write n/a if not applicable.)

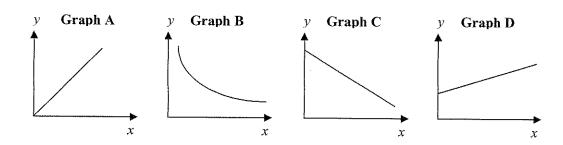
					· · · · · · · · · · · · · · · · · · ·			
a	b	c	d	e	f	g	h	i
η/a	www. J	(5)	races.	Ç.,	10%	· 3	C.J.	mente Langueto

(-1 each missing or incorrect answer)

Question 5 (1,1,1,1,2,2 marks)

(a)

The graphs below represent four different relationships between the variables x and y.



(i) Which graph(s) represent a direct proportion between x and y?

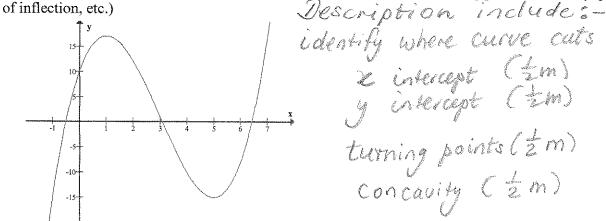


(ii) Which graph(s) represent an inverse proportion between x and y?

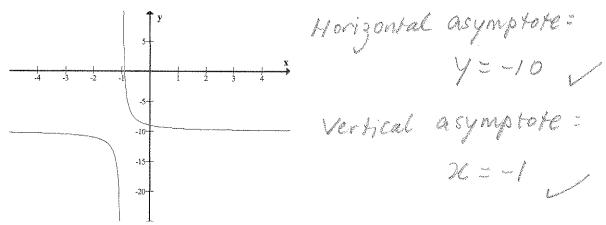
(iii) Suggest an equation that would represent:

(I) Graph B
$$y = \frac{k}{2c} \quad \text{where } k \text{ is a constant}$$

(b) For the cubic graph reproduced below, give a qualitative description of the function in the spaces below. (Note: calculations are *not* required.) However, you may wish to consider the following in your description of the function (i.e. intercepts, lines of symmetry, turning points, concavity, points of inflection, etc.)



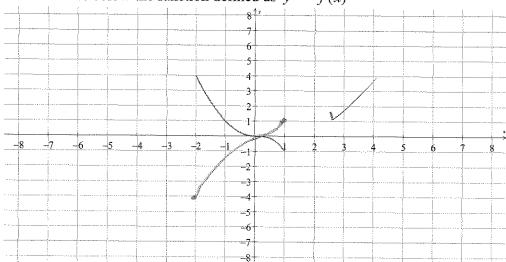
- VI for at least 3 correct statements 1 out of 2 for only 2 correct
- (c) The hyperbolic curve below has both horizontal and vertical asymptotes. State the value/s of x and y for which the asymptotes occur.



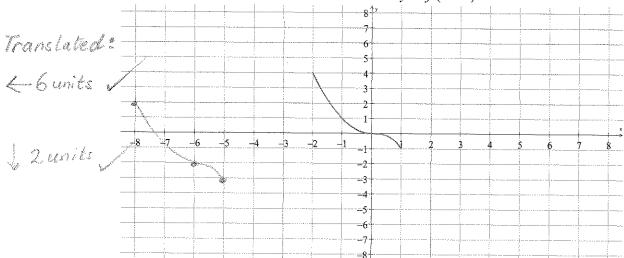
Question 6 (6 marks)

A function y = f(x) is constructed from parts of a cubic and parabolic functions as shown in the diagram below.

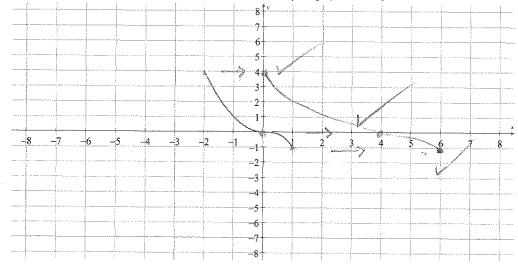
(a) Show on the axes below the function defined as y = -f(x)



(b) Show on the axes below the function defined as y = f(x+6) - 2



(c) Show on the axes below the function defined as y = f(0.5x - 2)



Question 7 (2,2,2,2,2,2 marks)

(a) A sequence is defined by $T_{n+1} = 3T_n$ with $T_1 = 5$. Determine the sixth term, T_6 .

Using Classpad

If by hand,

Showing understanding Concept (Im)

correct ans. (Im)

(b) A sequence has the recursive formula $T_{n+1} = (-1)^{n+1} 2 T_n$ with $T_1 = 3$. Determine the fifth term.

Using Classpad
Ts=48V

As above.

- A rocket is fired upwards and its initial speed is 147 metres per second. After 1 second its speed is 137.2 ms⁻¹, after the next second it is 127.4 ms⁻¹, and each subsequent second its speed is reduced by 9.8 ms⁻¹.
- (i) When does its speed drop to zero? By Classpad: The The 9.8

After 15 seconds (n=16)

(ii) What is the greatest height reached?

1/76 m

(d) Simplify, and hence evaluate:

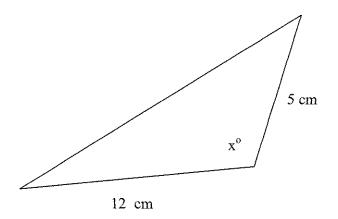
(i)
$$\frac{3^{n+2} + 18}{10 + 5 \times 3^n} = \frac{3^n \cdot 3^2 + 3^n \cdot 3}{5 \cdot 2 + 5 \cdot 3^n}$$

$$= \frac{3^n \cdot 3^2 + 3^n \cdot 3^n}{5 \cdot 2 + 5 \cdot 3^n}$$

Answers only: O.

Question 8 (2,2,2,marks)

(a) Determine the possible value/s of x (in degrees) given that the triangle has an area of 22.3 cm².



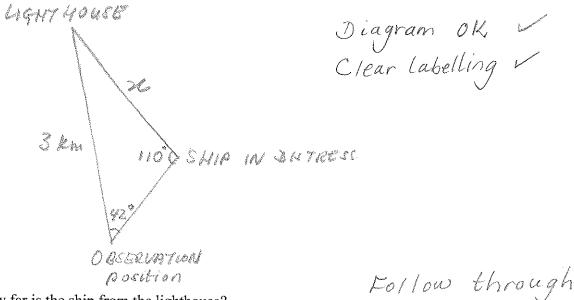
(Diagram not drawn to scale)

Using area formula:

$$22.3 = \frac{1}{2} (5)(12) \sin 36^{\circ}$$

$$22.3 = 48^{\circ} Vol 132^{\circ} Vol 132^{\circ}$$

- (b) A coastal observation position is known to be 3 km from a lighthouse. The coastguard in the observation position is in radio and visual contact with a ship in distress at sea. If the coastguard looks towards the lighthouse and then towards the ship these two directions make an angle of 42° with each other. If the captain on the ship looks towards the observation position and then towards the lighthouse these two directions make an angle of 110° with each other. (The ship, the lighthouse and the observation position may all be assumed to be at sea level).
- (i) Draw a clearly labelled diagram below to show the situation.



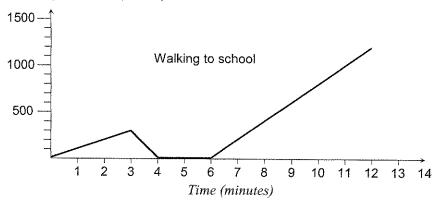
(ii) How far is the ship from the lighthouse?

 $\frac{2e}{\sin 42^{\circ}} = \frac{3}{\sin 110^{\circ}}$ $2e = 2.136 \approx 2.1 \text{ km}$ $\therefore \text{ Ship approx. } 2.1 \text{ km From Lighthouse}$

Question 9 (5 marks)

Amy started walking to school from home at 8.15 a.m. The graph below shows the distance from home against her walking time. This morning the walk to school was unusual.

Distance from home (metres)



(a) When did Amy realize that she had forgotten her homework?

At 8:18 am or 3 min. after she started walking

(b) How far was she from home?

300 m

(c) What did she do and how long did it take?

Ran home in 1 min

(d) What do you think happened in the two-minute period between 4 minutes and 6 minutes?

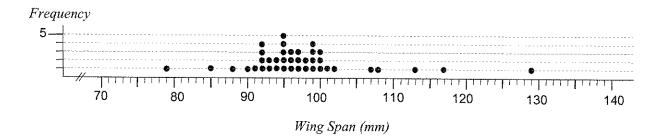
Looking for homework v

(e) How fast was she "walking" in the period a few minutes before arriving in school? (Give your answer in metres per minute.)

200 m per min

Question 10 (3,1,1,2 marks)

A scientist collects 40 butterflies of a particular species and measures the lengths of the wing span of each one. The lengths, to the nearest millimetre, are shown in the dot frequency diagram below.



(a) Calculate the mean and standard deviation for this set of lengths.

mean = 97.45 mm // s.d. = 8.35

- Accept 1 or 2.
- (b) What percentage of the lengths lie within one standard deviation of the mean (i.e. mean ± 1 s.d.)?

80% / RW

(c) Determine the interquartile range for this set of data?

1QR: 6.5

(d) Is the length of *one* particular butterfly with a wing span of 129 mm considered an outlier in this data set? Explain.

set? Explain. LQ M UQ.
93 96 99.5

1.5 x IQR = 9.75

99.5 + 9.75 = 109.25

Calculation

129 mm exceeds 1.5 x IQQ (109.25) } Explanation is an outlier.

Question 11 (1,2 marks)

In January last year, 150 black bream were tagged and released into the Swan River. In June the same year, during a fishing competition, 40 black bream were caught and 8 of them had tags. The tagged fish were released back into the river.

(a) Estimate the population of black bream in the Swan River in June last year.

Another fishing competition in January this year resulted in 60 black bream being caught. Fifteen of these had tags.

(b) Does the population of black bream in the Swan River appear to be increasing or decreasing? Justify your answer.

150 = 15 =7 x = 600 /

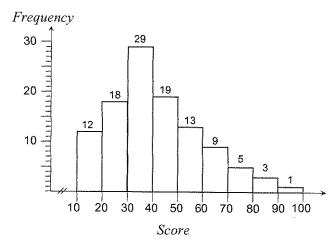
i. Population/appears to be

decreasing from 750 in Jan Last yr.

to 600 Jan this year. (1m).

Question 12 (4 marks)

The histogram below shows the distribution of scores achieved by students in a Mathematics exam.



Describe the distribution of marks in this exam.

Description includes.
Ventral tendency. Mean = 41.5

median & modal class lie in the 30 to 40

interval

Spread Store spread from 10 to 100, with

a range of 90

s. d. = 18.2

Skewness of skewed to right, 78 of 109 stores betw. 10 & 50, remaining 31 betw.

50 & 100

Just 9 of 109 score betw. 70 2100.

2

Question 13 (1,1,1,1,1,1 marks)

- (a) Determine
- $P(X \le 55)$ where $X \sim N(45, 10^2)$ (i)

0.8413

RW

(ii) $P(30 \le X \le 60)$ where $X \sim N(40, 81)$

0.8536

RW

- (b) The annual rainfall in equatorial Batam island, is normally distributed with a mean of 1300 mm and a standard deviation of 250 mm. According to this model, and assuming that the situation does not change, in every one hundred years how many years would you expect the rainfall to be:
- (i) less than 850 mm

0.0359 X 100 2 4 years /

(ii) more than 1600 mm

0.1151 (tm) X1002 12 year/

(iii) between 850 mm and 1600 mm

0.8490 X 100 & 85 years/ (c) In the town of Margaret River, the annual rainfall is normally distributed with a standard deviation of 380 mm. If there is only a 10% chance that the annual rainfall is greater than 2050mm, determine the mean annual rainfall.

